7 (6)

ASCHAM SCHOOL

FORM 6 - 2 UNIT MATHEMATICS TRIAL EXAMINATION

2000

July 2000

Time allowed: 3 Hours
Plus 5 minutes reading time.

Instructions

1. Attempt ALL questions
2. All questions are of equal value.
3. All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
4. Standard integrals are printed on page 9.
5. Board approved calculators may be used.
6. Answer each question in a *separate* writing booklet.

Question 1

(a) The price of a hamburger with 10% GST added is \$5.50. Find the original cost. [1]

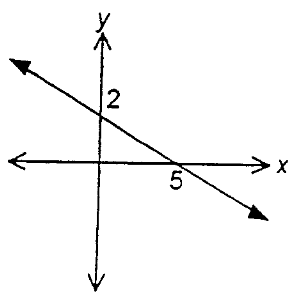
(b) What is the size of each angle in a regular pentagon? [1]

(c) If a $= 3.72\times10^{19}$ and b = 8.8×10^{-11} evaluate $\frac{a}{b}$ in scientific notation correct to 3 significant figures. [2]

(d) Solve $|3x-4|\le 4$ [2]

(e) Show that $\dfrac{1}{3-\sqrt{5}} - \dfrac{1}{\sqrt{5}-1}$ is a rational number. [2]

(f)



Find the equation of this line in general form. [2]

(g) Find the value of $5^3+5^2+5+\ldots\ldots\ldots$ [2]

Question 2 **Use a separate writing booklet.**

(a) Solve for m, $(2m-1)^2 = 25$ [2]

(b) A function is defined as $f(x)=\begin{cases}1-x^2 & \text{for } x\le 0\\ 5x-4 & \text{for } x>0\end{cases}$.

Evaluate $f(-2)+f(3)-f(0)$ [2]

(c) Factorise completely $2a^3 + 16$ [2]

(d) Simplify $\dfrac{x^2y^{-1}-y}{1-xy^{-1}}$ [2]

(e) Differentiate (i) $e^{2x}\cos x$

(ii) $\dfrac{\log x}{x}$

(ii) $\dfrac{1}{x} + (\log x)^4$ [4]

Question 3 **Use a separate writing booklet.**

(a) Simplify $\dfrac{\log_3 243}{\log_3 9}$ [2]

(b) Simplify $\left(1-\cos^2 X+\sin^2 X\right)\cot^2 X$ [2]

(c) The numbers p, p+2 and 9 are in geometric sequence. Find p. [2]

(d) If f and g are the roots of the equation $2x^2 - 6x + 7 = 0$ find the values of

(i) f + g

(ii) fg

(iii) $f^{-1} + g^{-1}$ [3]

(e) Find (i) $\displaystyle\int \frac{1+x}{x^2}\,dx$

(ii) $\displaystyle\int 6\cos(2-3x)\,dx$ [3]

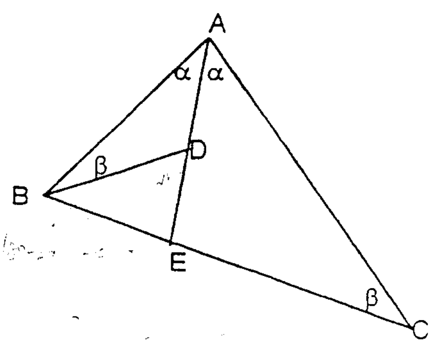
Question 4 **Use a separate writing booklet.**

(a) What is the domain and range of y = $-\sqrt{4-x^2}$ [2]

(b) Solve the equation tan2B = 1 for $-\pi \le B \le \pi$ [2]

(c) Find the value of m if the line 3x + 4y – 5 = 0 is parallel to the line (m-2)x – 2y +2 = 0. [2]

(d) In this diagram ∠BAE = ∠EAC = α and ∠ABD = ∠ECA = β.



(i) Copy the diagram.

(ii) Prove that BD = BE. [3]

(e) (i) Find the perpendicular distance of the line 3x+4y-10=0 from the origin.
(ii) Hence explain why the line 3x+4y-10=0 is a tangent to the circle $x^2 + y^2 = 4$. [3]

Question 5 **Use a separate writing booklet.**

(a) Simplify $\dfrac{36^{2n} \times 6^{2n-1}}{216^{2n}}$ [2]

(b) In an AP $u_2 = -2$ and $u_6 = 22$. Find u_{20} [3]

(c) Find the co-ordinates of the point on the curve $y = 3x^2 + 6x - 4$ at which the tangent is perpendicular to the line $x - 6y - 3 = 0$. [3]

(d) Write the equation of the parabola $y^2 - 6y + 25 = 8x$ in the form $(y-k)^2 = 4a(x-h)$ and find:

(i) the focal length

(ii) the co-ordinates of the focus

(iii) the equation of the directrix [4]

Question 6 **Use a separate writing booklet.**

Consider the curve given by $y = 3x - x^3$

(a) Show that the curve represents an odd function. [1]

(b) Find all stationary points and their nature. [3]

(c) Find any points of inflexion. [2]

(d) Sketch the curve for $-2 \le x \le 2.5$ [2]

(e) State the absolute minimum and maximum values in $-2 \le x \le 2.5$ [1]

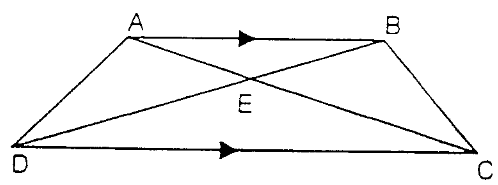
(f) Calculate the area bounded by the curve $y = 3x - x^3$ and the x-axis. [3]

Question 7 **Use a separate writing booklet.**

(a) Solve $1000^{2x} = 5$ to 2 significant figures. **[2]**

(b) The sum of the first n terms of a series is n^2 Write down the first term and the thirtieth term. [3]

(c)



The diagonals of the trapezium ABCD intersect at E

(i) Copy the diagram.

(ii) Prove that the triangles BAE and DCE are similar.

(iii) Hence prove that AE.DE = BE.CE **[4]**

(d) Shade the region $x^2 + (y - 1)^2 \le 1 \cap x + y < 1 \cap y \le 2^x$ **[3]**

Question 8 **Use a separate writing booklet.**

(a) Use the following table to approximately evaluate $\int_0^{0.4} f(p)\,dp$ to 3 significant figures by using Simpson's Rule.

p	0	0.1	0.2	0.3	0.4
f(p)	1	1.005	1.020	1.048	1.091

[2]

(b) AB is a chord of length 6cm of a circle, centre O. If the chord subtends an angle of 120° at O:

(i) Find the radius of the circle in exact form.

(ii) Find the length of minor arc AB in exact form. [4]

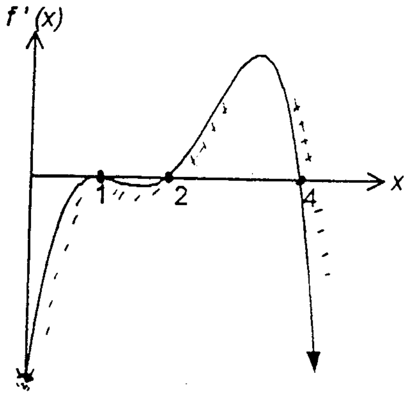
(c) Jane borrows \$5 000 from her father to pay for her Olympic tickets. They agree that Jane should pay interest of 1.5% every month and that she should pay her father back an instalment **every two months**.

(i) Letting $\$A_n$ be thé amount owing after n months and \$T be the value of each two-monthly instalment derive an expression, involving T, for the amount owing after 12 months.

(ii) Hence find the value of T to the nearest dollar if she pays back the loan after 2 years. [6]

Question 9 **Use a separate writing booklet.**

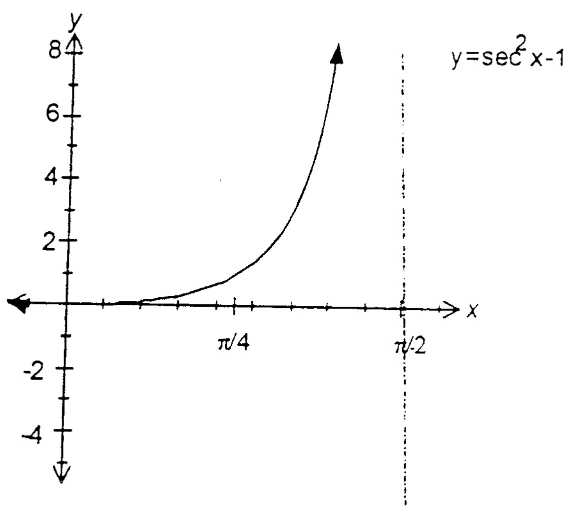
(a)



The diagram shows the graph of gradient function of the curve y = f(x). State the nature of the points at x=1, x=2 and x=4 where y = f '(x) cuts the x-axis.

[4]

(b)



(i) Copy the sketch shown into your answer booklet.

(ii) On the same axes sketch y = 8cosx – 1 for $0 \le x \le \frac{\pi}{2}$ and clearly mark the point of intersection, A, of the two graphs in the given domain.

(iii) Show that A= $\left(\frac{\pi}{3}, 3\right)$

(iv) Find the area bounded by y = $\sec^2 x$ - 1 and y = 8cosx – 1 and the line $x = \frac{\pi}{4}$ for the domain $\frac{\pi}{4} \le x \le \frac{\pi}{3}$. Give your answer in exact form.

[8]

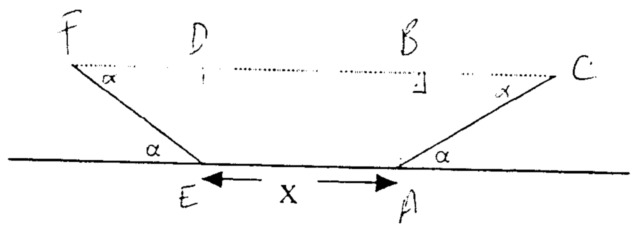
Question 10 **Use a separate writing booklet.**

(a) Show that the volume of the solid of revolution when the region bounded by $y = 3\log x$, the x and y axes and the line $y = \log 8$ is rotated about the y-axis is $\frac{9\pi}{2}$ cubic units. [5]

(b) A rectangular piece of metal is bent to form a gutter in the following manner. The left and right thirds are bent up so that each forms an angle α as shown in the figure below. The cross-section formed is a trapezium.

(i) Show that the cross-sectional area is given by $A = x^2(\sin\alpha + \sin\alpha\cos\alpha)$

(ii) What should α be in order that the cross-sectional area is a maximum?



Not to scale

[7]

End of exam

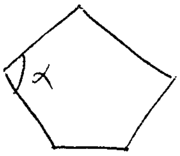
Ascham 2000 2U

Q1

A) $110\% x = \$5.50$

$10\% x = \$0.50$

$\therefore x = 10 \times \$0.50 = \underline{\$5}$

B) 

Sum of interior $\angle$s of a regular polygon $= 180°(n-2)$

Sum of $\angle$s in pentagon $= 180(3) = \underline{540°}$

$\therefore$ each $\angle = \frac{540°}{5} = \underline{108°}$

c) $\frac{a}{b} = \frac{3.72 \times 10^{19}}{8.8 \times 10^{-11}} = 4.23 \times 10^{29}$ (3.s.f)

D) $|3x-4| \leq 4$.

$-4 \leq 3x-4 \leq 4$

$0 \leq 3x \leq 8$

$\underline{0 \leq x \leq \frac{8}{3}}$

E) $\frac{1}{3-\sqrt{5}} - \frac{1}{\sqrt{5}-1} = \left(\frac{1}{3-\sqrt{5}} \times \frac{3+\sqrt{5}}{3+\sqrt{5}}\right) - \left(\frac{1}{\sqrt{5}-1} \times \frac{\sqrt{5}+1}{\sqrt{5}+1}\right)$

$= \left(\frac{3+\sqrt{5}}{4}\right) - \left(\frac{\sqrt{5}+1}{4}\right)$

$= \frac{3+\sqrt{5}-\sqrt{5}-1}{4} = \frac{2}{4} = \underline{\underline{\frac{1}{2}}}$

F) $y = -mx + b$

when $y=0$, $x=5$.

$\underline{0 = -5m + b.}$ — (1)

when $x=0$, $y=2$.

$2 = 0 + b$; $b = 2$ (sub into (1))

$-5m + 2 = 0$

$5m = 2$

$m = \frac{2}{5}$

$\therefore y = -\frac{2}{5}x + 2$; $y = \frac{-2x+10}{5}$

$\underline{5y + 2x - 10 = 0}$

G) $5^3+5^2+5+\dots$

GP with $r = \frac{5^2}{5^3} = \frac{1}{5}$

$S_\infty = \frac{5^3}{1-\frac{1}{5}} = \underline{156.25}$

Question 2

A) $(2m-1)^2 = 25$.

$4m^2 - 4m + 1 - 25 = 0$

$4m^2 - 4m - 24 = 0$.

$m^2 - m - 6 = 0$

$(m-3)(m+2) = 0$

$\underline{m=3}$ or $\underline{m=-2}$

B) $f(-2) + f(3) - f(0) = (1-4) + 5(3) - 4 - (1)$

$= -3 + 18 - 4 - 1$

$= 12 - 5 = \underline{7}$

C) $2a^3 + 16 = 2(a^3+8) = 2(a+2)(a^2-2a+4)$

D) $\dfrac{\frac{x^2}{y} - y}{1 - \frac{x}{y}} = \dfrac{x^2-y^2}{\cancel{y}} \times \dfrac{\cancel{y}}{y-x} = \dfrac{(x-y)(x+y)}{(y-x)}$

$= -(x+y)$

$= -x-y$

E) i.) $\frac{d}{dx} e^{2x}\cos x = \cos x(2e^{2x}) + e^{2x}(-\sin x)$

$= \underline{e^{2x}(2\cos x - \sin x)}$

ii.) $\frac{d}{dx} \frac{\log x}{x} = \frac{x(\frac{1}{x}) - \log x}{x^2} = \frac{1-\log x}{x^2}$

iii.) $\frac{d}{dx} \frac{1}{x} + (\log x)^4 = \frac{-1}{x^2} + 4(\log x)^3 \times \frac{1}{x}$

$= \underline{\frac{-1}{x^2} + \frac{4}{x}(\log x)^3}$

Question 3

A) $\dfrac{\log_3(3^5)}{\log_3(3^2)} = \dfrac{5\log_3 3}{2\log_3 3} = \dfrac{5}{2}$

B). $(1-\cos^2 x + \sin^2 x)\cot^2 x$

$= (1-\cos^2 x + \sin^2 x)\dfrac{\cos^2 x}{\sin^2 x}.$

$= 2\cancel{\sin^2 x} \times \dfrac{\cos^2 x}{\cancel{\sin^2 x}} = 2\cos^2 x$

c) ~~p+2~~ If it is a GP, then $\dfrac{p+2}{p} = \dfrac{9}{p+2}$

$(p+2)^2 = 9p$

$p^2+4p+4-9p=0$

$p^2-5p+4=0$

$(p-4)(p-1)=0$

$p=4$ or $\cancel{p=1}$

D) $P(x) = 2x^2 - 6x + 7 = 0$

i.) $f+g = \dfrac{-b}{a} = \dfrac{6}{2} = 3$

ii.) $fg = \dfrac{c}{a} = \dfrac{7}{2}$

iii) $\dfrac{1}{f} + \dfrac{1}{g} = \dfrac{g+f}{fg} = \dfrac{3}{\frac{7}{2}} = \dfrac{6}{7}$

E) i) $\displaystyle\int \frac{1+x}{x^2}\,dx = \int \frac{1}{x^2} + \frac{1}{x}\,dx$

$= \dfrac{x^{-1}}{-1} + \ln x + C$

$= \dfrac{-1}{x} + \ln x + C$

ii.) $\displaystyle\int 6\cos(2-3x)\,dx$

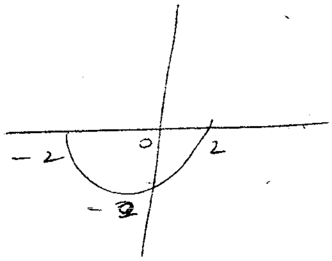
$= 6\displaystyle\int \cos(2-3x)\,dx$

$= 6\left(-\dfrac{1}{3}\sin(2-3x)\right) + C$

$= -2\sin(2-3x) + C$

Question 4

A) $y = -\sqrt{4-x^2}$



$D: -2 \le x \le 2$

$R: -2 \le y \le 0$

B) $\tan 2B = 1$ $\qquad -\pi \le B \le \pi$

$2B = \frac{\pi}{4}, \frac{5\pi}{4}, -\frac{3\pi}{4}, -\frac{7\pi}{4}$ $\qquad -2\pi \le 2B \le 2\pi$

$\frac{S \mid A}{T \mid C}$

$B = \frac{\pi}{8}, \frac{5\pi}{4}, -\frac{3\pi}{8}, -\frac{7\pi}{8}$

c) $3x + 4y - 5 = 0$

$y = \frac{5-3x}{4}$; $m_1 = -\frac{3}{4}$.

$(m-2)x - 2y + 2 = 0$.

$2y = (m-2)x + 2$

$y = \frac{(m-2)x+2}{2}$

$y' = \frac{m-2}{2}$ — m_2

since they are ∥, $m_1 = m_2$.

$\frac{m-2}{2} = -\frac{3}{4}$

$m - 2 = -\frac{6}{4}$

$m = 2 - \frac{6}{4} = \frac{1}{2}$ $\qquad \therefore m = \frac{1}{2}$

D) i) done

ii) $\angle DBE = 180° - 2\alpha - \beta - \beta$ ($\angle$ sum $\Delta = 180°$)

$= 180 - 2\alpha - 2\beta$

$\angle BDE = \alpha + \beta$ (exterior $\angle$ equals sum of interior opp $\angle$s)

$\angle BED = 180° - (180 - 2\alpha - 2\beta) - (\alpha + \beta)$ ($\angle$ sum $\Delta = 180$)

$= 2\alpha + 2\beta - \alpha - \beta = \alpha + \beta = \angle BDE$

$\therefore \angle BED = \angle BDE$

$\therefore BD = BE$ (base $\angle$ of isos ΔBDE are equal. $\therefore$ 2 sides are equal)

E) i.) $\perp$ distance $= \frac{|-10|}{\sqrt{9+16}} = \frac{10}{5} = 2$ units

ii.) The circle $x^2+y^2=4$
has a centre $(0,0)$ at the origin ~~and~~ a radius of 2 units.
Since the $\perp$ distance from origin to the line $= 2$ units,
$\therefore$ it is the radius of circle and touches ~~it~~ at the circumference.
$\therefore$ it is a tgt to the circle.

Question 5

A) $\frac{36^{2n} \times 6^{2n-1}}{216^{2n}} = \frac{6^{4n} \times 6^{2n-1}}{6^{6n}} = \frac{6^{6n-1}}{6^{6n}} = 6^{6n-1-6n} = 6^{-1} = \frac{1}{6}$

B) $u_2 = -2$; $-2 = a+d$ — ①
$u_6 = a+5d = 22$ — ②
② − ① $= a+5d-a-d = 24$
$4d = 24$
$d = 6$
$a = -8$

$u_{20} = a+19d$
$= -8+19(6) = 106$

c) $y = 3x^2+6x-4$
$\frac{dy}{dx} = 6x+6$ — m_1

$x-6y-3=0$
$6y = x-3$; $y = \frac{x-3}{6}$; $\frac{1}{6}$ — m_2

$\frac{1}{6}(6x+6) = -1$ ($m_1 m_2 = -1$)
$6x+6 = -6$
$6x = -12$
$x = -2$, $y = 3(4)+6(-2)-4 = -4$
$\therefore (-2,-4)$

D) $y^2 - 6y + 25 = 8x.$

$(y^2 - 6y + 9) + 25 - 9 = 8x$

$(y-3)^2 + 16 = 8x$

$(y-3)^2 = 8(x-2)$

i) focal length: $4a = 8$

$a = 2$

ii) vertex $= (2,3)$

focus $= (4,3)$

iii) Directrix $x = 0$ ie the y-axis

Question 6

A) $y = 3x - x^3$

Let $y = f(x)$

$f(x) = 3x - x^3$

$f(-x) = -3x - (-x)^3 = -3x + x^3$

$-f(x) = -(3x - x^3) = -3x + x^3$

Since $f(-x) = -f(x)$, $f(x)$ is an odd function.

B) $f'(x) = 3 - 3x^2$

At stat. pts, $f'(x) = 0$

$3 - 3x^2 = 0$

$1 - x^2 = 0$; $x = 1$ or $x = -1$

$y = 2$ $\quad$ $y = -2$

Determine nature

$f''(x) = -6x$

At $(1,2)$, $f''(x) = -6 < 0$ $\therefore$ $(1,2)$ is a max. t.pt

At $(-1,-2)$, $f''(x) = 6 > 0$ $\therefore$ $(-1,-2)$ MIN t.pt

C) At possible pts of inflexion, $f''(x) = 0$

$-6x = 0$

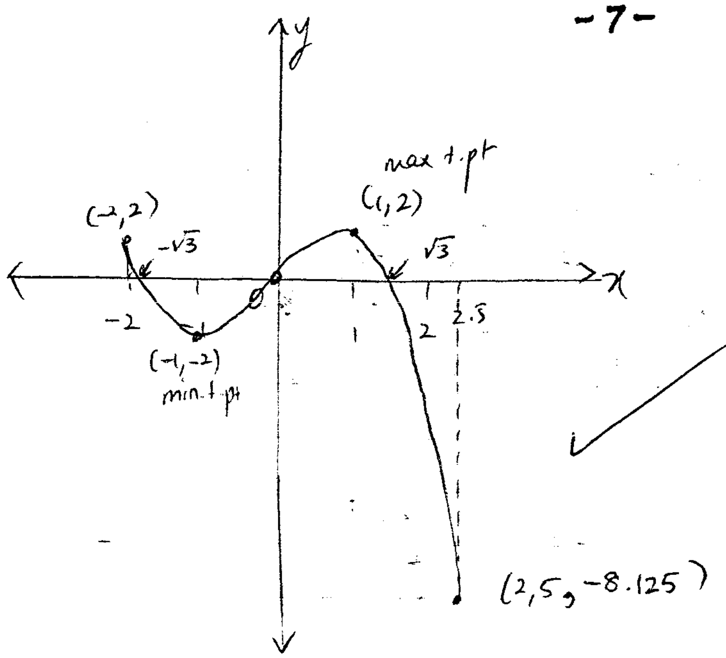
$x = 0$; $y = 0$

Check for change in concavity.

x	0^-	0	0^+
y''	>0	0	<0

$\therefore$ since there is a change in concavity, $(0,0)$ is a pt of inflexion

D)



E) Absolute minimum value = -8.125
" max " = 2

F) ~~Area~~ $y = x(3-x^2)$
It cuts the x axis at $x=0$, $x=\sqrt{3}$ and $x=-\sqrt{3}$

$$\text{Area} = \int_{-2}^{-\sqrt{3}} 3x - x^3 dx + \left|\int_{-\sqrt{3}}^{0} 3x - x^3 dx\right| + \int_{0}^{\sqrt{3}} 3x - x^3 dx + \left|\int_{\sqrt{3}}^{2.5} 3x - x^3 dx\right|$$

$$= \left(\frac{3x^2}{2} - \frac{x^4}{4}\right)_{-2}^{-\sqrt{3}} + \left|\left(\frac{3x^2}{2} - \frac{x^4}{4}\right)_{-\sqrt{3}}^{0}\right| + \left(\frac{3x^2}{2} - \frac{x^4}{4}\right)_{0}^{\sqrt{3}} + \left|\left(\frac{3x^2}{2} - \frac{x^4}{4}\right)_{\sqrt{3}}^{2.5}\right|$$

$$= \left(\frac{9}{2} - \frac{9}{4} - 6 + 4\right) + \left|\frac{9}{4} - \frac{9}{2}\right| + \left(\frac{9}{2} - \frac{9}{4}\right) + \left|\frac{3 \times 2.5^2}{2} - \frac{39.0625}{4} - \frac{9}{2} + \frac{9}{4}\right|$$

$= 0.25 + 2.25 + 2.25 + 2.640625$
$= \underline{7\ 390625\ u^2}$

Q7

A) $1000^{2x} = 5$
$2x \ln 1000 = \ln 5$
$x = \frac{\ln 5}{2 \ln 1000} = 0.12$ (2 s.f)

B) $S_1 = 1^2$ ∴ the first term = 1
$S_2 = 2^2$
$S_{29} = 29^2$ The 30^{th} term $T_{30} = S_{30} - S_{29}$
$S_{30} = 30^2$ $= 30^2 - 29^2 = \underline{59}$

∴ $T_1 = 1$, and $T_{30} = 59$

c) i.) done

ii.) In ΔBAE and ΔDCE,

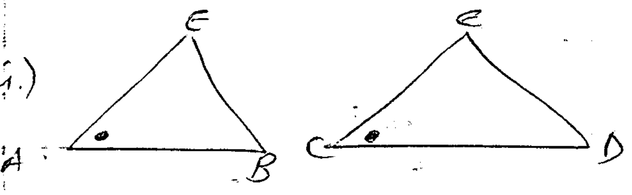
① $\angle AEB = \angle DEC$ (vert. opp $\angle$ equal) ✓

② $\angle BAE = \angle ECD$ (alt $\angle$ on $AB \parallel DC$) ✓

③ $\angle ABE = \angle EDC$ (" ")

$\therefore$ $\Delta BAE \,|||\, \Delta DCE$ (equiangular) ✓

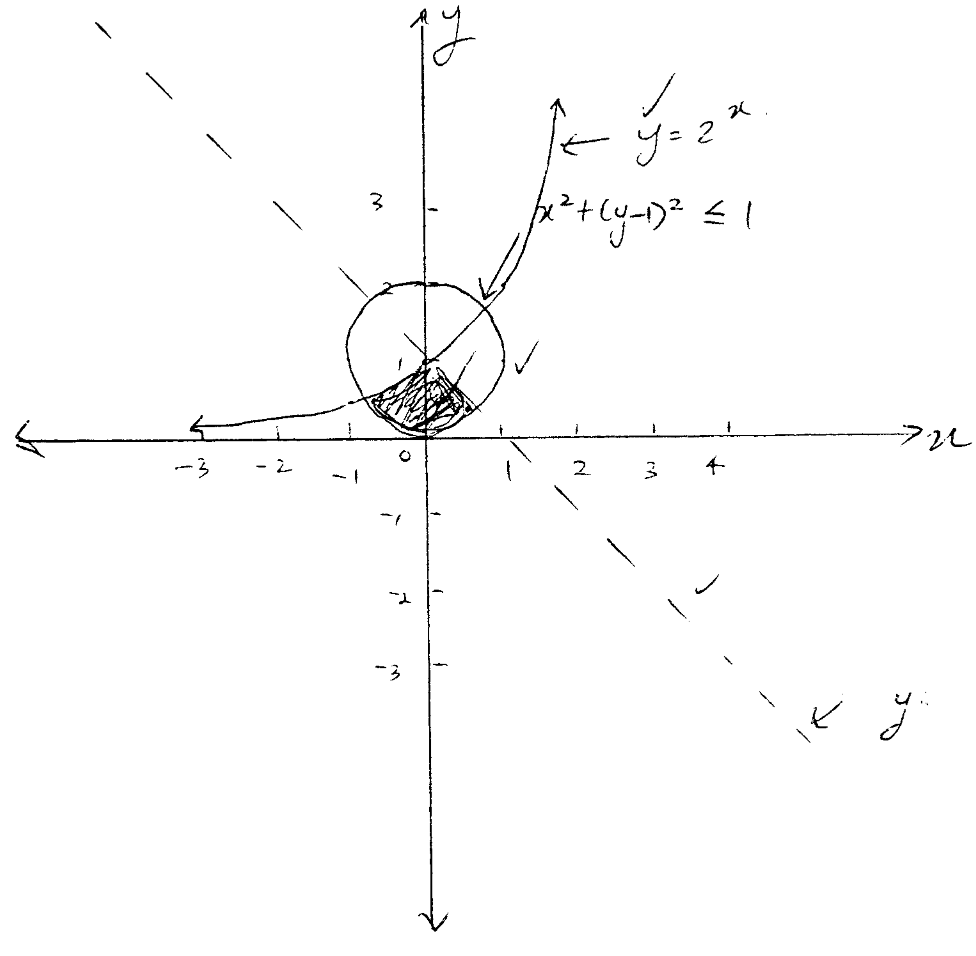
iii.)



$\frac{AE}{CE} = \frac{BE}{DE}$ (corresponding sides of $|||\,\Delta$s are in same proportion) ✓

$\therefore AE \times DE = CE \times BE$ ✓

D)

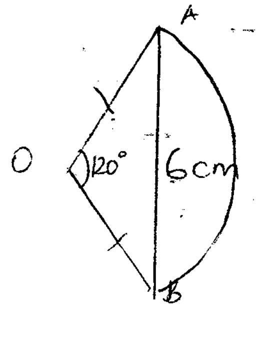


Question 8

A) $\int_0^{0.4} f(p)\,dp \doteqdot \frac{0.1}{3}\left(1+4(1.005)+2(1.020)+4(1.048)+1.091\right)$ ✓

$= \underline{0.411}$ (3 s.f) ✓

B)



i.) $120^\circ = \frac{2\pi}{3}$ ✓

$\angle OAB = \angle OBA = \frac{180-120}{2} = 30^\circ$

($\angle$ sum $\Delta = 180^\circ$; base $\angle$ of isos ΔOAB are equal)

In ΔOAB, $\frac{\sin\frac{2\pi}{3}}{6} = \frac{\sin\frac{\pi}{6}}{OA}$ ✓

$OA = \frac{6(0.5)\times 2}{\sqrt{3}}$

$OA = \frac{6\sqrt{3}}{3} = \underline{2\sqrt{3}\text{ cm}}$ ✓

ii.) length of arc AB

$= 2\sqrt{3}\left(\frac{2\pi}{3}\right)$

$= \underline{\frac{4\sqrt{3}\pi}{3}\text{ cm}}$ ✓

C) i.) $A_1 = \$5{,}000(1.015)$ $\qquad$ A_n = amount owing after n months.

$A_2 = \$5{,}000(1.015)^2 - T$

$A_3 = \$5{,}000(1.015)^3 - T(1.015)$

$A_4 = \$5{,}000(1.015)^4 - T(1.015)^2 - T$ ✓

$A_4 = \$5000(1.015)^4 - T(1+1.015^2)$

$A_n = \$5000(1.015)^n - T(1+1.015^2 + \quad 1.015^{n-2})$

$A_{12} = \$5000(1.015)^{12} - T(1+1.015^2 + \ldots\ 1.015^{10})$ ✓

ii.) Note $A_{24} = 0$.

$A_{24} = \$5000(1.015)^{24} - T(1+1.015^2 + \ldots\ 1.015^{22})$

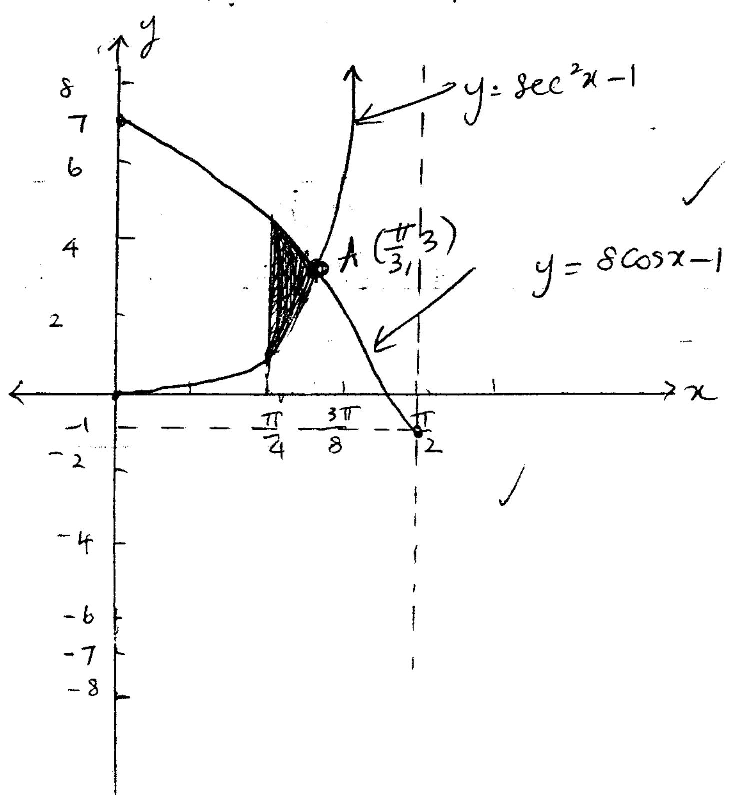
$0 = \$5000(1.015)^{24} - T\left(\frac{(1.015^2)^{12} - 1}{1.015^2 - 1}\right)$ ✓

$T\left(\frac{1.015^{24}-1}{1.015^2-1}\right) = \$5000(1.015)^{24}$; $T = \underline{\$503}$ (to nearest dollar) ✓

Question 9

A) At $x = 1 \leftarrow$ decreasing pt. of inflexion ✓
$x = 2 \leftarrow$ min. t.pt ✓
$x = 4 \leftarrow$ max t.pt ✓

B)



iii) $y = 8\cos x - 1$ — ①
$y = \sec^2 x - 1$ — ②
to find A, solve ① and ② simult.
From ① — $\cos x = \frac{y+1}{8}$ (sub into ②)

$$y = \left(\frac{8}{y+1}\right)^2 - 1$$

$$y = \frac{64 - y^2 - 2y - 1}{y^2 + 2y + 1}$$

$$y(y^2+2y+1) = 64 - y^2 - 2y - 1$$
$$y^3 + 2y^2 + y - 64 + y^2 + 2y + 1 = 0.$$
$$y^3 + 3y^2 + 3y - 63 = 0$$
$$(y+1)^3 - 64 = 0$$
$$(y+1)^3 = 64$$
$$y + 1 = 4$$
$\underline{y = 3}$ ✓; $\cos x = \frac{4}{8} = \frac{1}{2}$ $\therefore \underline{x = \frac{\pi}{3}}$

$\therefore A = \left(\frac{\pi}{3}, 3\right)$

iv.) Area $= \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} 8\cos x - 1 - \sec^2 x + 1 \, dx$

$= \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} 8\cos x - \sec^2 x \, dx = \left(8\sin x - \tan x\right)_{\frac{\pi}{4}}^{\frac{\pi}{3}}$

$= 8\sin\frac{\pi}{3} - \tan\frac{\pi}{3} - 8\sin\frac{\pi}{4} + \tan\frac{\pi}{4}$

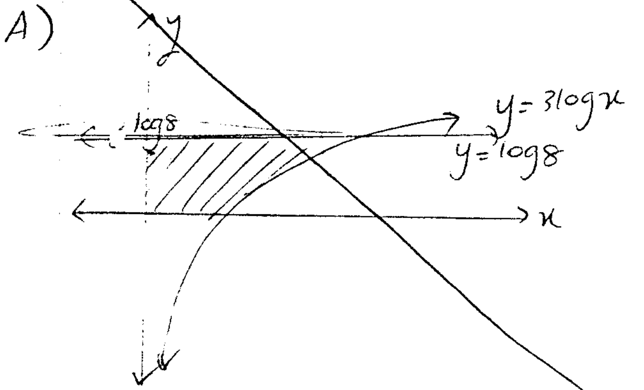
$= 8\left(\frac{\sqrt{3}}{2}\right) - \sqrt{3} - 8\left(\frac{1}{\sqrt{2}}\right) + 1$

$= 4\sqrt{3} - \sqrt{3} - 4\sqrt{2} + 1$

$= (3\sqrt{3} - 4\sqrt{2} + 1)$ units^2

Question 10

A)



$y = 3\log x$

$\log x = \frac{y}{3}$ OR $\frac{\ln x}{\ln 10} = \frac{y}{3}$

$x = 10^{\frac{y}{3}}$ $\quad \ln x = \frac{y\ln 10}{3}$

$x = e^{\frac{y\ln 10}{3}}$

$V = \pi \int_0^{y\log 8} \left(10^{\frac{y}{3}}\right)^2 dy$

$V = \pi \int_0^{y\log 8} 10^{\frac{2y}{3}} dy$ $\quad \ln 10 = \frac{\ln x}{y} \cdot 3$

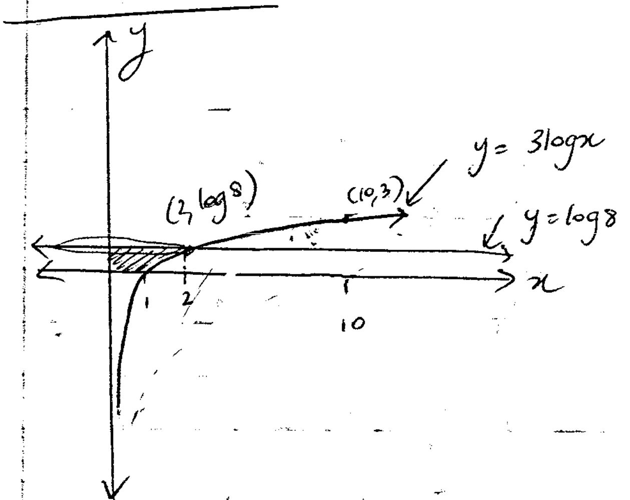
$V = \pi \int_0^{\log 8} \left(e^{\frac{y\ln 10}{3}}\right)^2 dy$

$= \pi \int_0^{\log 8} e^{\frac{2y\ln 10}{3}} dy = \pi \left(\frac{3}{2\ln 10} e^{\frac{2y\ln 10}{3}}\right)_0^{\log 8}$

$= \pi \left(\frac{3}{2\ln 10} e^{\frac{2\ln 10 \log 8}{3}} - \frac{3}{2\ln 10}\right)$

$= \pi \left(\frac{12 - 3}{2\ln 10}\right) = \frac{9\pi}{2\ln 10}$

Question 10



$\ln x = \frac{y \ln 10}{3}$

$x = e^{\frac{y\ln 10}{3}}$

$V = \pi \int_0^{\log 8} e^{\frac{2y\ln 10}{3}}\, dy$

$= \pi \left(\frac{1}{\frac{2\ln 10}{3}} e^{\frac{2y\ln 10}{3}} \right)_0^{\log 8}$

$= \pi \left(\frac{3}{2\ln 10} e^{\frac{2\ln 10 \log 8}{3}} - \frac{3}{2\ln 10} \right)$

$= \pi \left(\frac{12 - 3}{2\ln 10} \right) = \frac{9\pi}{2\ln 10}$

$y = 3\log x$ ← Stephanie, This means $\log_e$. I know they shouldn't do this!

$\log x = \frac{y}{3}$

$\frac{\ln x}{\ln 10} = \frac{y}{3}$

$\ln x = \frac{y \ln 10}{3}$

$\ln x = \frac{\ln 10^y}{3}$

$x = e^{\frac{\ln 10^y}{3}}$

$\therefore x = e^{y/3}$

$Vol. = \pi \int_0^{\ln 8} (e^{y/3})^2\, dy$

$= \pi \int_0^{\ln 8} e^{\frac{2y}{3}}\, dy$

$= \pi . \frac{3}{2} \left[e^{\frac{2y}{3}} \right]_0^{\ln 8}$

$= \frac{3\pi}{2} \left[e^{\frac{2}{3}\ln 8} - e^0 \right]$

$= \frac{3\pi}{2} [4 - 1]$

$= \frac{9\pi}{2}$.

B) 1.) In ΔABC, $\frac{\sin\alpha}{AB} = \frac{\sin(90-\alpha)}{BC}$

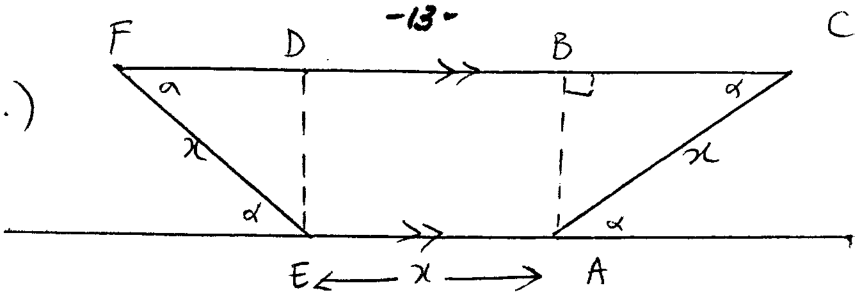
$\frac{\sin\alpha}{AB} = \frac{\cos\alpha}{BC}$

$AB = \frac{BC\sin\alpha}{\cos\alpha} = BC\tan\alpha$ $\therefore AB = BC\tan\alpha$

$BC = AB\cot\alpha$. similarly $FD = AB\cot\alpha$

Area of cross section $= \frac{1}{2}(x + x + 2AB\cot\alpha)\cdot x$.

B) i.)



In ΔBCA, $\frac{\sin 90}{x} = \frac{\sin\alpha}{AB}$; $AB = x\sin\alpha$. ✓

$\tan\alpha = \frac{AB}{BC} = \frac{x\sin\alpha}{BC}$; $\underline{BC = x\cos\alpha}$ ✓

Total area $= 2\left(\frac{1}{2} \times x\cos\alpha \times x\sin\alpha\right) + x\,(x\sin\alpha)$

$= x^2\cos\alpha\sin\alpha + x^2\sin\alpha$ ✓

$= \underline{x^2(\sin\alpha + \sin\alpha\cos\alpha)}$ ✓

ii.) $\frac{dA}{d\alpha} = x^2(\cos\alpha + \cos 2\alpha)$ ✓

For max/min A, $\frac{dA}{d\alpha} = 0$

$x^2(\cos\alpha + 2\cos^2\alpha - 1) = 0$

$2\cos^2\alpha + \cos\alpha - 1 = 0$

$(2\cos\alpha - 1)(\cos\alpha + 1) = 0$

$\cos\alpha = \frac{1}{2}$ or $\cos\alpha = -1$ ✓

$\alpha = \frac{\pi}{3}$ ✓ $\alpha = \pi$ But $\alpha \neq \pi$ because $0 < \alpha < \frac{\pi}{2}$.

Determine nature

$\frac{d^2A}{d\alpha^2} = x^2(-\sin\alpha - 2\sin 2\alpha)$

When $\alpha = \frac{\pi}{3}$, $\frac{d^2A}{d\alpha^2} = x^2\left(-\frac{\sqrt{3}}{2} - \sqrt{3}\right) < 0$

$\therefore$ area is max. when $\alpha = \frac{\pi}{3}$ ✓